stage 11:
foundations for multiplication

Big Idea: Repeated Equal Groupings

The Repeated Equal Groupings big idea builds upon the Parts-to-Whole idea. With Repeated Equal Groupings, the whole is not only broken into parts but broken into a specific number of parts and each part is of equal size. For example, there are 30 students in the class (the whole). The teacher divides the class into groups of 6 (the equal groups). These equal groups are repeated 5 times to equal the whole.

Repeated equal groupings is the big idea that underlies multiplication and division. Multiplication consists of taking a part (the multiplicand) and repeating it a certain number of times (the multiplier) to equal the whole (the product). Division consists of a whole (the dividend) that is partitioned into a certain number of equal groups (the divisor) that is equal to the size of the parts (the quotient).

Why are Repeated Equal Groupings Important?

It is possible for students to memorize multiplication facts without understanding what they mean. Students can also use skip counting to correctly solve multiplication problems without appreciating the significance of how they arrived at the correct answer. In addition to being able to immediately recall multiplication and division facts, students also need to be able to understand what these operations mean.

Students who understand equal groupings have a better chance of memorizing multiplication and division number relationships because they have a conceptual basis to support their learning. For example, they are more likely to see the connections between, 7 x 4, 4 x 7, 28 ÷ 7, and 28 ÷ 4 . Equal groupings is also fundamental to fractions. Students who are only relying on memorized facts and skip counting strategies may have difficulty understanding the meaning of fractions and how they build upon multiplication and division.

Stages 11 and 13 develop an understanding of grouping and partitioning by building on the Parts-to-Whole concepts established in Stages 3, 6, 8, and 10. They reinforce the concept of repeated groupings - that multiplication represents repeated addition and division represents repeated subtraction. Previous Symphony Math Stages help students develop their conceptual understanding of what these operations mean, and then help students learn the number relationships through systematic practice and evaluation.

Stages 11 and 13 address number relationships with products and dividends up to thirty. This approach uses the repeated addition model of multiplication and the repeated subtraction model of division. In turn, the models help students understand the connection between addition and multiplication and between subtraction and division.

Stage 11 Learning Progression

Concept	Standard	Example	Description
11.1: Skip Counting	2.NBT.2	2, 4, 6, ?, ?	Students count on from any number and say the next few numbers that come afterwards, counting by 2s, 5s, 10s, 20s, 50s, and 100s. Skip counting is introduced to help students work towards multiplication concepts. It is not yet true multiplication because they don't keep track of the number of groups counted, yet, they can experience that when counting by 2s, 5s, 10s, 20s and so on, they are counting by groups with that amount in each group.
11.2: Adding 2s, 3s, and 4s	2.OA.4	2 + 2 + 2 = ?	Using the array formation, students find the sum of equal addends up to 5 addends.
11.3: Equal Groups	2.OA.4	'Create 4 groups of 2'	Focusing on story problems, students show that they can model quantities presented in a given story. They use Symphony Math tools to show that they understand groups versus the numbers per group. No calculations are necessary since the goal is to show an understanding and interpretation of problem quantities. In time, learning how to model story problem quantities with a visual model will help students solve these problems.

Using the Guided Practice Materials

When students struggle with a concept, you will see suggeted Guided Practice materials in your HELP data view of your Symphony Math Dashboard. These materials provide extended practice using the Multiples Ways of Knowing from the Symphony Math program:

Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	1) Read the story presented at the top of the page. 2) Create a number model of the full solution. 3) Write the number sentence that matches the model.

Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.

Stage Checkpoint

When students complete a Stage, they come to a Checkpoint. Checkpoints provide students an opportunity to draw and create models in their offline journals. When they are done, you can evaluate their work and make sure they are ready to move on in the Symphony Math curriculum. Here is a short checklist to guide your evaluation:

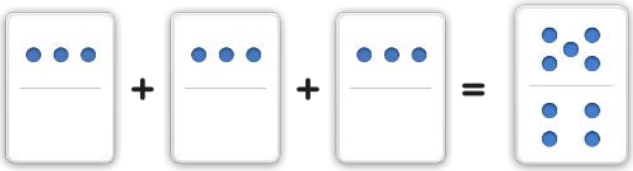
- ☐ All Checkpoint steps have been attempted
- ☐ Student demonstrates understanding of models
- ☐ Solutions are correct, or intervention has led to self-correction
- ☐ Student can talk about their work using mathematical language

Students' math understanding is evolving. Their work at Checkpoints gives them a chance to transfer their new skills to another context (an offline journal). It is also an opportunity for reflection. Be sure to allow students to create more pages if they need it. When you are confident they are ready, accept the student checkpoint from your Symphony Dashboard.

Name: _____ Date: _____

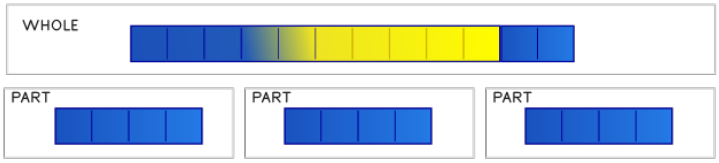
Stage II: Teacher Reference

Dot Cards:



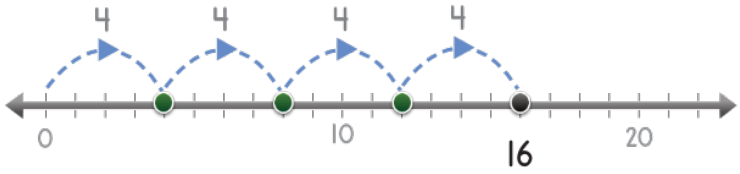
3 + 3 + 3 = 9

Number Bars:



12 = 4 + 4 + 4

Number Line:



4 + 4 + 4 + 4 = 16

Create Your Own:

Here are ____ equal groups of ____:

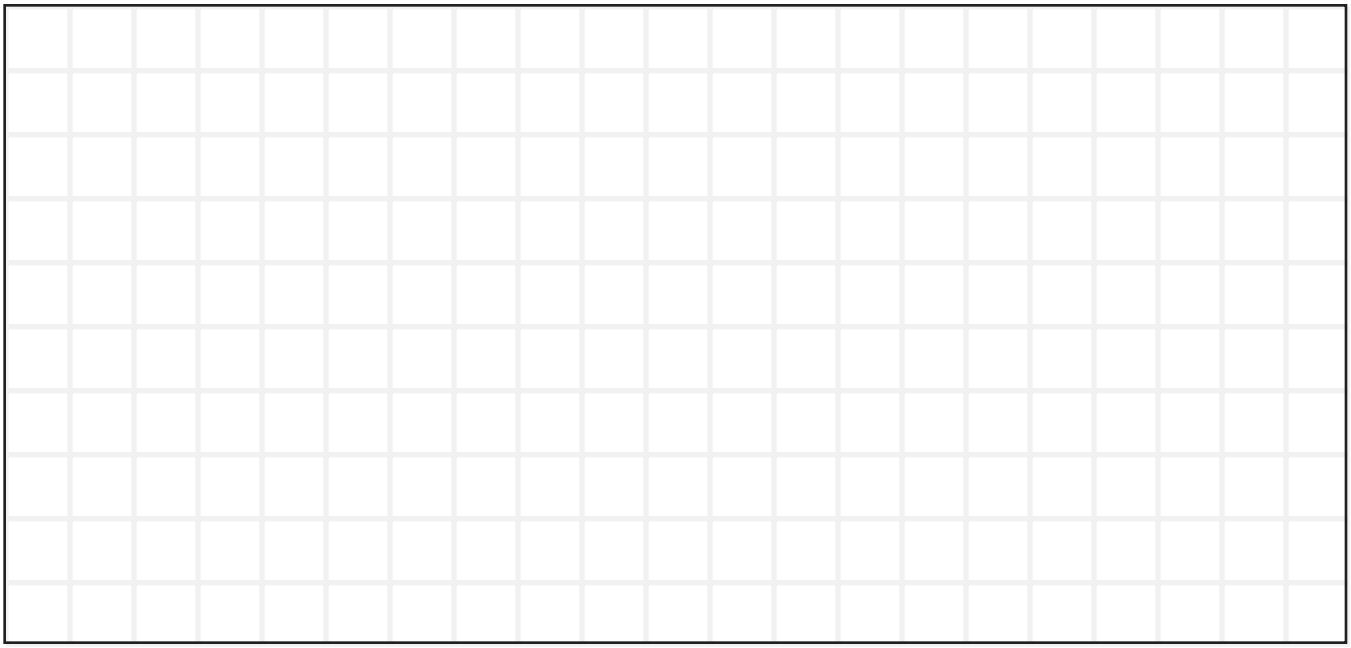
Dot Cards:

Number Line:

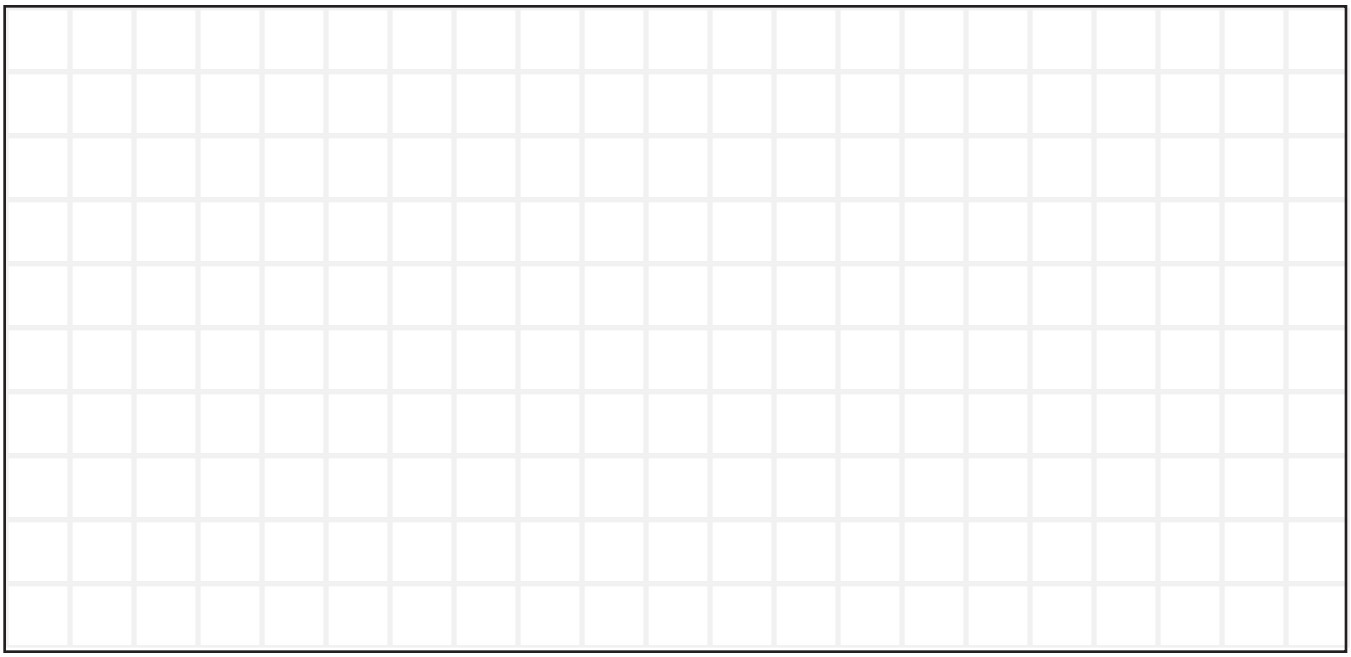
Name: _____ Date: _____

Stage II: Foundations for Multiplication

Dot Cards:



Number Bars:



Name: _____ Date: _____

Checkpoint

Number Line:

MY OWN: