stage 22:
expanded mode multiply/divide

Big Idea: Repeated Equal Groupings and Hierarchical Groupings

Repeated equal groupings is the big idea that underlies multiplication and division. Multiplication consists of taking a part (the multiplicand) and repeating it a certain number of times (the multiplier) to equal the whole (the product). Division consists of a whole (the dividend) that is partitioned into a certain number of equal groups (the divisor) that is equal to the size of the parts (the quotient).

Hierarchical Groupings is the idea that amounts can be grouped into a system of sets and subsets. We count 11 objects and group them into 1 ten and 1 one, or we can call them 11 ones. Seventy five represents 7 tens and 5 ones. When an understanding of hierarchical groupings is combined with repeated equal groupings, students can begin to understand the relationship between 4 x 7, 4 x 70, and 4 x 700.

Students in Stage 22 see a task- 7 x 36- presented vertically next to an area model. They label one side of the area model with 7, and the 36 on the other side can be partitioned into tens and ones, or, 30 and 6. In Symphony Math, students are familiar with multiplying and dividing by multiples of ten (Stage 16), and with place value partitions; in Stage 22, they partition the number and use the distributive property to compute in steps 7 x 30 + 7 x 6. The resulting partial products appear, step by step, under the vertical presentation of the problem as they go along. Using the visual area models, students are able to make sense of and multiply and divide large quantities by adding partial products. Story problems are presented and students work them using these same models.

Why are Hierarchical Groupings Important?

In Stage 16, students experienced how numbers grow multiplicatively when using 1s, 10s, and 100s. Stage 22 consolidates ideas that have been used in every previous Stage and provides an entry point for the multiplication and division of large quantities using multiples of ten and expanded notation: harder problems can be broken down into their parts, and made accessible. Thus is the groundwork laid for the standard algorithm students confront. Without an understanding of hierarchical groupings, the computation processes in the standard algorithm may be procedural and lack sense-making. In Symphony Math, use of the distributive property and expanded notation give students a foundation for the complex task of multiplying and dividing larger quantities.

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Stage 22 Learning Progression

Concept	Standard	Example	Description
22.1: 1-Digit x 2-digit	4.NBT.5	7 x 18 = ?	Students see a vertically presented multiplication problem next to an area model. They label the 1-digit factor along one side length of the box and decompose the 2-digit factor into tens and ones along the other side. An animated line accompanies this partition into tens and ones. Using the distributive property, students then compute partial products and watch the products automatically appear in the original vertically presented equation as they go along. These stacked partial products are added together for the final answer.
22.2: Division with Remainders	4.NBT.6	25 ÷ 8 = ? R ?	Students' understanding of multiplication is able to be transferred to division with the use of the same visual area model. The problem 13 ÷ 4 = ? is a division problem, but the model with the grid is the same one they see when they multiply. In the division example, the dividend is inside the box, the divisor is along one side length, and the quotient, or missing factor, for the other side must be determined. In this way, the problem may be thought of as a missing factor problem and the connection between multiplication and division is explicit. When there is a remainder, students can see the extra part of a box that is left over; the part that eventually can make a new whole.
22.3: 1-Digit x 3-Digits	4.NBT.5	8 x 106 = ?	Students see a vertically presented multiplication problem next to an area model. As in Stage 22.1, partial products appear stacked in the vertical problem as they go along, thus laying the groundwork for the standard algorithm students see in later grades.
22.4: Division Missing Factor and Divisor	4.NBT.6	102 ÷ 17 = ?	Students see a division problem presented next to an area model. Advancing the idea that multiplication and division are inverse operations, in 22.4, the model for computing the quotient relies on the model for multiplication.
22.5: 2-Digits x 2-Digits	4.NBT.5	26 x 21 = ?	Using the distributive property and multiples of tens and hundreds, students have entry into a potentially difficult problem.The visual area model, and the stacked partial products appearing under the vertically presented multiplication, offer students sensible access to the standard algorithm when it appears later.

Using the Guided Practice Materials

When students struggle with a concept, you will see suggeted Guided Practice materials in your HELP data view of your Symphony Math Dashboard. These materials provide extended practice using the Multiples Ways of Knowing from the Symphony Math program:

Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	1) Read the story presented at the top of the page. 2) Create a number model of the full solution. 3) Write the number sentence that matches the model.

Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.

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Stage Checkpoint

When students complete a Stage, they come to a Checkpoint. Checkpoints provide students an opportunity to draw and create models in their offline journals. When they are done, you can evaluate their work and make sure they are ready to move on in the Symphony Math curriculum. Here is a short checklist to guide your evaluation:

- ☐ All Checkpoint steps have been attempted
- ☐ Student demonstrates understanding of models
- ☐ Solutions are correct, or intervention has led to self-correction
- ☐ Student can talk about their work using mathematical language

Students' math understanding is evolving. Their work at Checkpoints gives them a chance to transfer their new skills to another context (an offline journal). It is also an opportunity for reflection. Be sure to allow students to create more pages if they need it. When you are confident they are ready, accept the student checkpoint from your Symphony Dashboard.

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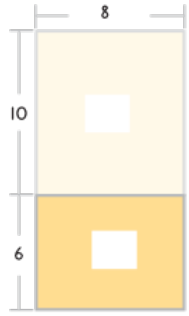
Name: _____ Date: _____

Stage 22: Teacher Reference

Example A:

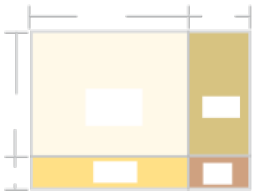
Complete the grid model and number sentence:



$$\begin{array}{r} 16 \\ \times\ \ 8 \\ \hline \\ + \\ \hline \\ \end{array}$$

Example B:

Complete the grid model and number sentence:

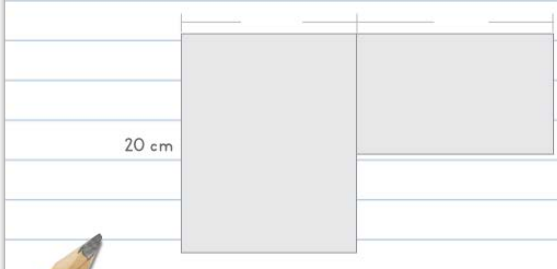


$$\begin{array}{r} 34 \\ \times\ 57 \\ \hline \\ + \\ \hline \\ \end{array}$$

Create Your Own:

The shape below is missing some dimensions.

Estimate the missing lengths and use the grids to find the area of the shape.



Area: _____

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Name: _____ Date: _____

Stage 22: Expanded Mode X÷

Example 1:

Example 2:

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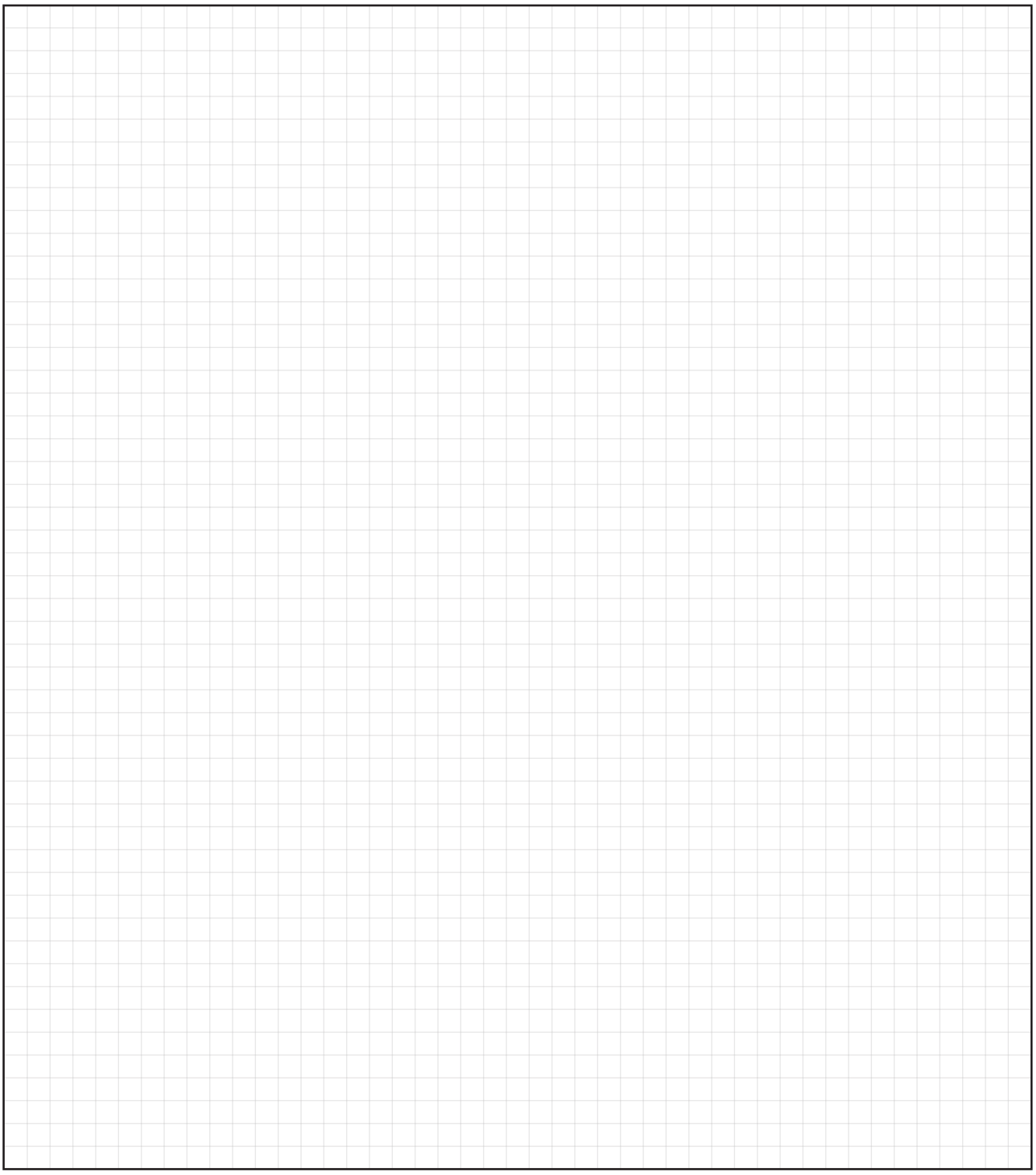
Name: _____ Date: _____

Checkpoint

MY OWN:



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